Job Assignment using Hungarian Algorithm

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Abstract—Management and Job Assignment has, over the years, become the critical ingredient in organizational theory and practice This paper reviews and emphasizes on optimally solving the assignment problem and thereby assigning the right job to the right employee based on their technical knowledge as well as managerial skills, and various important qualities such as work experience, ability to delegate, innovation and commitment. In all industries, Recruitment and Job Assignment is a highly necessary task, and deep care and precaution should be taken before making the final decision. In this paper, we have developed ways and parameters to scrutinize a resume and decide which candidate is suitable for which position in the company, depending on various pre-decided parameters. This task can be accomplished using Hungarian Algorithm. It uses the concepts of Optimization (which falls under the domain of Operations Research) to show how the different theories and lines of research fit together to bring out the optimized results for the task being performed.

Keywords:

Job Assignment Hungarian Algorithm Operations Research OPT: Optimization Principles And Techniques

1. INTRODUCTION

An organizational system consists of various subunits. The most efficient approach to optimize the performance of a system is to consider the various subunits as a combined single system. In some real time cases, integrating all the subunits as a single system will make the problem solving process more complicated, because of its size and various other constraints. Under such situation, it is inevitable to optimize the performance of each subunit.

2. OPTIMIZATION ALGORITHMS

Operations Research is a scientific approach to problem solving for executive decision making which requires the formulation of mathematical, economic and statistical models. Operational research (OR) encompasses a wide range of problem-solving techniques and methods applied with the goal of improved decision-making and efficiency, such as simulation, neural networks, queuing theory, mathematical optimization, decision analysis, expert systems and the analytic hierarchy process. Most of these techniques involve construction of mathematical models that attempt to describe the system. Because of the computational and statistical nature of most of the fields, OR has strong ties to computer science and analytics.

Optimization Principles and Techniques falls under the domain of **"Operations Research".** In the simplest case, an optimization problem consists of maximizing or minimizing a real function by choosing input values from within an allowed set and calculating the value of the function. More generally, optimization includes finding "best available" values of some objective function given a set of constraints.

3. NEED FOR ASSIGNING RIGHT JOB TO THE RIGHT PERSON

Job Assignment is the most influential and critical element of all businesses. Assigning the right job to the right employee based on their technical knowledge, managerial skills, and various important qualities such as work experience, ability to delegate, innovation, commitment, communication skills, etc, will not only help in a better understanding of the company's objectives and goals, but also provide necessary tools and skills to inspire and impact the various subunits, thereby allowing the firm to run competently and smoothly. In all industries, recruitment and Job Assignment is a very essential task, and deep care and precaution should be taken before assigning the candidates to the different levels of the organizations.

4. PROBLEM

To understand this need for assigning the right job to the right employee, we consider an example of a company, Starks Organization, which basically needs to assign one candidate each to 4 Middle-level Management Positions viz, General Manager, Project Manager, Department Manager and Branch Manager. For this purpose, the highest level authorities have been observing quite a few of their top notch employees, from which they would like to promote four candidates. They shortlisted four of the best employees, Allen, Bob, Cedric and Derek. They also predetermined the Technical Knowhow, Management Qualifications and certain vital qualities such as work experience, ability to delegate, innovation, communication skills and commitment, etc, required for each position. Each of the candidates had to give an aptitude test and submit their resumes.

5. SOLUTION

The **Assignment Problem** is one of the fundamental combinational optimization problems in the branch of operations research. It is a special kind of Transportation Problem in which each source should have the capacity to fulfill the demand of any of the destinations.

General Form: There are n number of agents and n number of tasks. Any agent can be assigned to perform any task, with some predetermined capability (in percentage) of fulfilling the task, that may vary depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task and exactly one task to each agent in such a way that the efficiency of the agent in the assigned task is maximized.

This problem of Job Assignment can be solved using Hungarian Algorithm. The **Hungarian method** is a combinational optimization algorithm that solves the assignment problem in polynomial time. It basically consists of 2 phases. In the first phase, row reductions and column reductions are carried out. In the second phase, the solution is optimized on iterative basis.

Following are the Steps to solve Assignment Problem using Hungarian Algorithm:

1. Determine the cost table from the problem.

- If no. of sources is equal to no. of destinations, go to step 3.

- If no. of sources is not equal to no. of destinations, go to step2.

2. Add a dummy source or dummy destination, so that the table becomes a square matrix. The table entries of the dummy source or destinations are always zero.

3. Row Reduction: Find the smallest element in each row of the given matrix and then subtract the same from each element of the row.

4. Column Reduction: In the reduced matrix obtained in the step 3, find the smallest element of each column and then subtract the same from each element of that column.

5. In the modified matrix obtained in the step 4, search for the optimal assignment:

- Examine the rows successively until a row with a single zero is found. En-rectangle this row and cross off (X) all other zeros in its column. Continue in this manner until all the rows have been taken care of.

- Repeat the procedure for each column of the reduced matrix.

- If a row and/or column has two or more zeros and one cannot be chosen by inspection then assign arbitrary any one of these zeros and cross off all other zeros of that row/column.

- Repeat (a) through (c) above successively until the chain of assigning or cross ends.

6. If the number of assignment is equal to n (the order of the cost matrix), an optimum solution is reached. If the number of assignment is less than n, go to the next step.

7. Draw the minimum number of horizontal and/or vertical lines to cover all the zeros of the reduced matrix. 8. Develop the new revised cost matrix as follows:

- Find the smallest element of the reduced matrix not covered by any of the lines. - Subtract this element from all uncovered elements and add the same to all the elements laying at the intersection of any two lines.

9. Go to step 6 and repeat the procedure until an optimum solution is attained.

5.1 Tables

Table 1: Data

| | Gen Mngr | Proj Mngr | Dept Mngr | Branch Mngr |
|---|-------------|--------------|--------------|----------------|
| А | 82 | 50 | 60 | 21 |
| В | 34 | 22 | 84 | 34 |
| С | 93 | 60 | 34 | 28 |
| D | 85 | 69 | 35 | 80 |

| | Gen Mngr | Proj Mngr | Dept Mngr | Branch Mngr |
|---|-------------|--------------|--------------|----------------|
| А | 11 | 43 | 33 | 72 |
| В | 59 | 71 | 9 | 59 |
| С | 0 | 33 | 59 | 65 |
| D | 8 | 24 | 58 | 13 |

Table 3: Row Reduction

| | Gen Mngr | Proj Mngr | Dept Mngr | Branch Mngr |
|---|-------------|--------------|--------------|----------------|
| А | 0 | 32 | 22 | 61 |
| В | 50 | 62 | 0 | 50 |
| С | 0 | 33 | 59 | 65 |
| D | 0 | 16 | 50 | 5 |

Table 4: Column Reduction

| | Gen Mngr | Proj Mngr | Dept Mngr | Branch Mngr |
|---|-------------|--------------|--------------|----------------|
| А | 0 | 16 | 22 | 56 |
| В | 50 | 46 | 0 | 45 |
| С | 0 | 17 | 59 | 60 |
| D | 0 | 0 | 50 | 0 |

Table 5: Iteration 1

| | Gen Mngr | Proj Mngr | Dept Mngr | Branch Mngr | |
|---|-------------|--------------|--------------|----------------|---|
| А | 0 | 16 | 22 | 56 | Х |
| В | 50 | 46 | 0 | 45 | |
| С | 0 | 17 | 59 | 60 | Х |
| D | 0 | 0 | 50 | 0 | |
| | Х | | | | |

Table 6: Iteration 2

| | Gen Mngr | Proj Mngr | Dept Mngr | Branch Mngr |
|---|-------------|--------------|--------------|----------------|
| | А | В | C | D |
| А | 0 | 0 | 6 | 40 |
| В | 66 | 46 | 0 | 45 |
| С | 0 | 1 | 43 | 44 |
| D | 16 | 0 | 50 | 0 |

Table 7: Assignment

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|----------------------|------|--------------|--------------|--------|--|
| | Gen | Proj Mngr | Dept Mngr | Branch | |
| | Mngr | Mngr | Mngr | Mngr | |
| А | 82 | 50 | 60 | 21 | |
| В | 34 | 22 | 84 | 34 | |
| С | 93 | 60 | 34 | 28 | |
| D | 85 | 69 | 35 | 80 | |

6. RESULTS

Candidate 'A' is best suited for the designation **Project Manager**, candidate 'B' for the designation **Department Manager**, candidate 'C' for the designation **General Manager** and candidate 'D' for the designation **Branch Manager**.

7. CONCLUSIONS

7.1. Advantages And Disadvantages

Initially, the accepted solution of the Assignment problem was the determination of all possible permutations, resulting in O(n!) complexity. Obviously, this solution scaled very poorly with large data sets, making very big implementations not practical. With the advent of the Hungarian algorithm, polynomial complexity of O(n4) was attained, later optimized to O(n3). Due to the iterative implementation of the matrix approach used by Hungarian algorithm, costly recursive calls were avoided, instead performing the basic operation of row and column reduction, via a sequence of loops. This method also facilitates usage of a two-dimensional array to represent the data set.

Considering the alternative, an O(n!) method, the Hungarian algorithm is practically efficient and simple way of solving the assignment problem. The only problem is that large data arrays will have a large storage overhead, but it makes up for that with its lightening speed. As such, it has always been an effective and commonly implemented approach since its inception.

7.2. Applications

As a general optimization algorithm, Hungarian algorithm finds frequent use in economics, and mostly in the calculation and narrowing down of production functions via optimization. Additionally, it has been used in solving problems of transportation, profit maximization and expenditure minimization for units distributed geographically (taxi services being a prime example). Hence, any problem that has any kind of assignment of a weighted element in a permutation could be solved efficiently via the Hungarian algorithm.

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